Math 4 $\boxed{\text{Key}}$
1-6 Sequences

I. Notation for Sequences

A sequence, like the one you found above for the minimum number of moves for the Tower of Hanoi problem, is an **ordered list of numbers**. The sequence is named with a letter and a subscript, for example a_n. The letter is used to name the sequence, and the subscript ("n") refers to the position of a number in the sequence. For instance, a_4 would refer to the 4th term in sequence a.

Given the sequence $\{0, 1, 1, 2, 3, 5, 8, 13, 21, 34, ...\}$

Find the following terms: $a_4 =$ 2 ; $a_1 =$ 0 ; $a_9 =$ 21 ; $a_{6+1} =$ 8

a_7

II. Formulas for Sequence

Some sequences have formulas that generate them. Formulas can be either explicit or recursive. An **explicit formula** is a formula that allows direct computation of any term in the sequence. A **recursive formula** requires the computation of all previous terms in order to find the value of term a_n (NEXT-NOW formulas are examples of recursive sequences).

Consider the sequence of positive even numbers $\{2, 4, 6, 8, 10, 12, 14, 16, ...\}$

The *explicit formula* for this sequence is $a_n = 2n$

The *recursive formula* for this sequence is $\begin{cases} a_1 = 2 \\ a_n = a_{n-1} + 2 \end{cases}$

Note that for the recursive formula, you must know the first term, then you define the rest of the terms in relation to the first term.

Write out the first five terms of the following sequences:

$g_n = 2n - 1$ 1, 3, 5, 7, 9

$k_i = i^2 - 1$ 0, 3, 8, 15, 24

$\begin{cases} q_1 = 3 \\ q_n = 2 \cdot q_{n-1} \end{cases}$ 3, 6, 12, 24, 48

$\begin{cases} b_1 = 4 \\ b_{n+1} = 2 \cdot (b_n) - 3 \end{cases}$ 4, 5, 7, 11, 19

III. **Arithmetic Sequences**

An ***arithmetic sequence*** is a sequence in which the difference between two consecutive terms is the same. The *common difference* is found by subtracting any term from its succeeding term. The n^{th} *term* (a_n) of an arithmetic sequence with first term a_1 and the common difference is d is given by the following formula: $a_n = a_1 + (n-1)d$

A. Name the first five terms of each arithmetic sequence defined below. An example is given.

Example: $a_1 = 2, d = 3 \rightarrow$ 2, 5, 8, 11, 14

1. $a_1 = 4, d = 3$ 4, 7, 10, 13, 16

2. $a_1 = 7, d = 5$ 7, 12, 17, 22, 27

3. $a_1 = -\frac{4}{5}, d = 1$ $-4/5, \frac{1}{5}, \frac{6}{5}, \frac{11}{5}, \frac{16}{5}$

B. Name the next four terms of each of the following arithmetic sequences:

Example: 5, 9, 13 17, 21, 25, 29

+4 +4

-6

1. 21, 15, 9, 3, -3, -9, -15

+3

2. 11, 14, 17, 20, 23, 26, 29

+3.8

3. 9.9, 13.7, 17.5, 21.3, 25.1, 28.9, 32.7

C. Use $a_n = a_1 + (n-1)d$ to find the n^{th} term of the sequence (n is given)

Example: $a_1 = 7, d = 3, n = 14$

$a_{14} = 7 + (14-1)3$

$a_{14} = 7 + 13 \cdot 3 = \boxed{46}$

1. $a_1 = -1, d = 10, n = 25$

$a_{25} = -1 + (25-1) \cdot 10 = \boxed{239}$

2. $a_1 = 2, d = \frac{1}{2}, n = 8$ $a_8 = 2 + (8-1)\frac{1}{2} = \boxed{5.5}$

3. $a_1 = 27, d = 16, n = 23$

$a_{23} = 27 + (23-1)16 = \boxed{379}$

D. Find the indicated term in each arithmetic sequence:

Example: a_{12} for -17, -13, -9, . . . $\xrightarrow{+4}$

$a_{12} = -17 + (12-1)4 = \boxed{27}$

1. a_{21} for 10, 7, 4 $\xrightarrow{-3}$ $a_{21} = 10 + (21-1)(-3) = \boxed{-50}$

2. a_{10} for 8, 3, -2 $\xrightarrow{-5}$ $a_{10} = 8 + (10-1)(-5) = \boxed{-37}$

3. a_{12} for $\frac{3}{4}, \frac{3}{2}, \frac{9}{4}$ $\xrightarrow{+3/4}$ $a_{12} = \frac{3}{4} + (12-1)(\frac{3}{4}) = \boxed{9}$

E. Which term?

Example: Which term of -2, 5, 12, . . . is 124?

$a_1 = -2$
$d = 7$
$a_n = 124$
$n = ?$

$124 = -2 + (n-1)7$

$\boxed{n = 19}$

1. Which term of -3, 2, 7, . . . is 142? $\xrightarrow{+5}$

$142 = -3 + (n-1)5 \longrightarrow \boxed{n = 30}$

2. Which term of 7, 2, -3, . . . is -28? $\xrightarrow{-5}$

$-28 = 7 + (n-1)(-5)$ $\boxed{n = 8}$

F. Find the missing terms of the following arithmetic sequences:

Example: 55, <u>70</u>, <u>85</u>, <u>100</u>, 115

a_1 a_2 a_3 a_4 a_5

$115 = 55 + (5-1)d$

$d = 15$

1. -10, <u>-8.8</u>, <u>-7.6</u>, <u>-6.4</u>, <u>-5.2</u>, -4

a_6

$-4 = -10 + (6-1)d$

$d = \frac{6}{5}$

2. 2, <u>5</u>, <u>8</u>, <u>11</u>, <u>14</u>, <u>17</u>, 20

a_7

$20 = 2 + (7-1)d$

$d = 3$

II. **Geometric Sequences**

A ***geometric sequence*** is a sequence in which each term after the first is found by multiplying the previous term by a constant. In any geometric sequence, the *constant or common ratio* is found by dividing any term by the previous term. The n[th] term (a_n) of a geometric sequence with first term a_1 and constant ratio r is given by the formula $a_n = a_1 \cdot r^{n-1}$

A. Determine which of the following sequences are geometric. If it is a geometric sequence, find the common ratio.

Example: $\overset{\times 5}{4, 20, 100, 500}$ Geometric; r = 5

1. 7, 14, 28, 56, . . . geometric; r = 2
2. 2, 4, 6, 8, . . . not geometric; arithmetic w/ d = 2
3. 3, 9, 27, 54, . . . not geometric; $27 \cdot 3 = 81 \neq 54$
4. 9, 6, 4, $\frac{8}{3}$ geometric; $r = \frac{2}{3}$

B. Find the next two terms for each geometric sequence:

Example: 729, 243, 81, . . . $r = \frac{1}{3}$ 27, 9

1. 20, 30, 45, . . . r = 1.5 $\boxed{67.5, 101.25}$
2. 90, 30, 10, . . . $r = \frac{1}{3}$ $\boxed{\frac{10}{3}, \frac{10}{9}}$
3. 2, 6, 18, . . . r = 3 $\boxed{54, 162}$

C. Find the first four terms of each geometric sequence described below:

Example: $a_1 = \frac{3}{2}, r = 2$

1. $a_1 = 3, r = -2$ 3, −6, 12, −24
2. $a_1 = 12, r = \frac{1}{2}$ 12, 6, 3, $\frac{3}{2}$
3. $a_1 = 27, r = -\frac{1}{3}$ 27, 9, 3, 1.

D. Find the nth term of each geometric sequence described below:

Example: $a_1 = 4, n = 3, r = 5$ $a_3 = 4 \cdot 5^{3-1} = \boxed{100}$

1. $a_1 = 4, n = 3, r = 2$ $a_3 = 4 \cdot 2^{3-1} = \boxed{16}$

2. $a_1 = 2, n = 5, r = 2$ $a_5 = 2 \cdot 2^{5-1} = \boxed{32}$

3. $a_1 = 32, n = 6, r = -\frac{1}{2}$ $a_6 = 32 \cdot (-\frac{1}{2})^{6-1} = \boxed{-1}$

E. Find the missing terms of the following geometric sequences:

Example: 3, 6, 12, 24, 48 a_5 $48 = 3 \cdot r^{5-1}$

$\boxed{r = 2}$

1. 243, 81, 27, 9, 3 a_5 $3 = 243 \cdot r^{5-1}$

$r = \frac{1}{3}$

F. Given the geometric sequence $\frac{1}{4096}(2)^{n-1}$, which term in the sequence is 16777216? *Hint: you will need to use logarithms to solve this problem.*

$16777216 = \frac{1}{4096} 2^{n-1}$

$2^{n-1} = (16777216)(4096)$

$(n-1)\log 2 = \log [16777216 \cdot 4096]$

$\boxed{n = 37}$

F. Given the **geometric sequence** $t_n = \frac{1}{4096}(2)^{n-1}$, which term in the sequence is 16777216?

$$\left(16777216 = \frac{1}{4096}(2)^{n-1}\right) \cdot 4096$$

$$68719476736 = 2^{n-1} \longrightarrow \log_2 68719476736 = n-1$$

$$36 = n-1$$

$$n = \boxed{37^{th} \text{ term}}$$

V. **Finding Equations**

A. Find an explicit and recursive equation for the following sequences:

1. $\{2,5,8,11,...\}$

Arith.

Explicit Formula:

$a_n = 2 + 3(n-1)$

Recursive Formula

$$\begin{cases} a_1 = 2 \\ a_n = a_{n-1} + 3 \end{cases}$$

or $a_{n+1} = a_n + 3$

2. $\{7,21,63,...\}$

Geo.

Explicit Formula:

$a_n = 7(3)^{n-1}$

Recursive Formula

$$\begin{cases} a_1 = 7 \\ a_n = a_{n-1} \cdot 3 \end{cases}$$

or

$a_{n+1} = a_n \cdot 3$

3. $\{28, 24.542, 21.084, 17.626, \ldots\}$

Arith

Explicit

$$a_n = 28 - 3.458(n-1)$$

Recursive

$$\begin{cases} a_1 = 28 \\ a_n = a_{n-1} - 3.458 \end{cases}$$

4. $\left(\frac{50}{21}, \frac{100}{147}, \frac{200}{1029}, \ldots\right)$

Geo.

Explicit

$$a_n = \frac{50}{21}\left(\frac{2}{7}\right)^{n-1}$$

Recursive

$$\begin{cases} a_1 = \frac{50}{21} \\ a_n = a_{n-1} \cdot \frac{2}{7} \end{cases}$$